Perceptual Color Volume
Measuring the Distinguishable Colors of HDR and WCG Displays

INTRODUCTION

Metrics exist to describe display capabilities such as contrast, bit depth, and resolution, but none yet exist to describe the range of colors produced by high-dynamic-range and wide-color-gamut displays. This paper proposes a new metric, *millions of distinguishable colors* (MDC), to describe the range of both colors and luminance levels that a display can reproduce.

This new metric is calculated by these steps:

1) Measuring the XYZ values for a sample of colors that lie on the display gamut boundary
2) Converting these values into a perceptually-uniform color representation
3) Computing the volume of the 3D solid

The output of this metric is given in millions of distinguishable colors (MDC). Depending on the chosen color representation, it can relate to the number of just-noticeable differences (JND) that a display (assuming sufficient bit-depth) can produce.

MEASURING THE GAMUT BOUNDARY

The goal of this display measurement is to have a uniform sampling of the boundary capabilities of the display. If the display cannot be modeled by a linear combination of RGB primaries, then a minimum of 386 measurements are required to sample the input RGB cube (9x9x6). Otherwise, the 386 measurements may be simulated through five measurements (Red, Green, Blue, Black, and White).

We recommend the following measuring techniques:
- For displaying colors, use a center-box pattern to cover 10% area of the screen
- Using a spectroradiometer with a 2-degree aperture, place it one meter from the display
- To exclude veiling-glare, use a flat or frustum tube mask that does not touch the screen

CONVERTING TO A PERCEPTUALLY-UNIFORM COLOR REPRESENTATION

After the XYZ values of the gamut boundary are collected either by measurement or simulation, the values must be converted to a perceptually-uniform representation. This paper describes conversion to two color representations: CIE L*a*b* [1] and IC_TC_P [2]. IC_TC_P is recommended because it is a more perceptually-uniform color representation (see Appendix A for details). CIE L*a*b*, however, may be preferred because it is more widely used.

When reporting the final value, specify which representation is used.

XYZ to IC_TC_P *(Recommended)*

Calculate LMS
$$L = 0.3593X + 0.6976Y - 0.0359Z$$
$$M = -0.1921X + 1.1005Y + 0.0754Z$$
$$S = 0.0071X + 0.0748Y + 0.8433Z$$

Apply the PQ Function (ST 2084 [3])
$$L'M'S' = EOTF_{PQ}^{-1}(LMS)$$

Calculate IC_TC_P
$$I = 0.5L' + 0.5M'$$
$$C_T = (6610L' - 13613M' + 7003S')/4096$$
$$C_P = (17933L' - 17390M' - 543S')/4096$$

Scale to JND's
$$IC_TC_P = [I,C_T,C_P] * [720,360,720]$$

XYZ to CIE L*a*b*

$$L* = 116 * f(Y/Y_n) - 16$$
$$a* = 500 * [f(X/X_n) - f(Y/Y_n)]$$
$$b* = 200 * [f(Y/Y_n) - f(Z/Z_n)]$$

where:

$$f(t) = \begin{cases} t^{1/3} & t > (6/29)^3 \\ t(1/3)(29/6)^2+(16/116) & otherwise \end{cases}$$

(X_n, Y_n, Z_n) is the measured white point

Note: Each axis of IC_TC_P is scaled [4] (from the original in ITU-R BT.2100 [5]) to convert the color representation from an efficient interchange format to a perceptually-uniform JND space.

COMPUTING THE 3D VOLUME

Now that we have measurements of the color-volume boundary and because we know the location of the points, it is fairly simple to compute the volume of the solid described by the vertices. (Refer to [6] for further explanation.) The 3D-volume calculation process is broken down into these steps (see the example Matlab code in Appendix B):

1) Calculating the tessellation
2) For each triangle, computing the signed volume to the origin
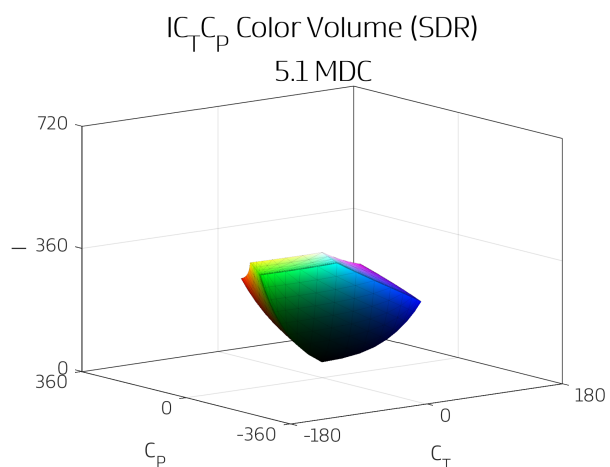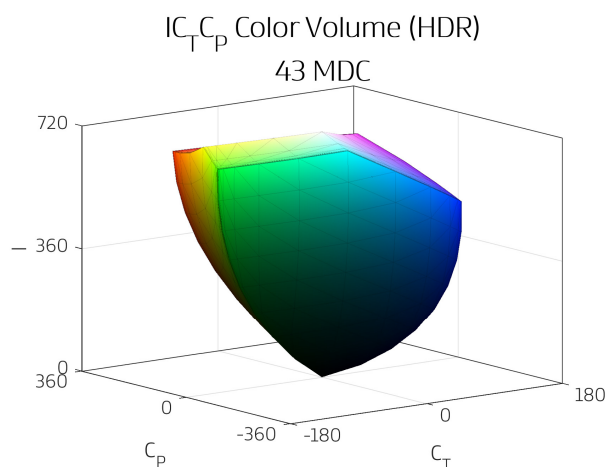3) Summing the signed volumes and dividing by one million

MDC REPORTING

After the 3D volume has been computed, the resulting number is typically in units of millions of distinguishable colors (MDC). Because the relative volumes are determined by the color representation used to perform the calculations, specify the color representation with the reported number.

Although reporting the raw MDC number is useful, it may be of interest to compare the MDC between displays. Below are two examples using the IC_TC_P MDC calculation:

- Example 1: %HDR (using BT.2100 color primaries and White=10000, Black=0 cd/m^2)
- Example 2: %SDR (using BT.709 [7] color primaries and White=100, Black=0.1 cd/m^2)

White Level (cd/m^2)	Black Level (cd/m^2)	Color Primaries	MDC	%HDR	%SDR
10,000	0	BT.2100	43	100%	843%
100	0.1	BT.709	5.1	12%	100%
1000	0.05	DCI P3 [8]	18	42%	353%

The percentage may be helpful, but should be reported only in addition to the raw MDC value. Based on application, other reference displays may also be used for comparison.

REFERENCES

[1] CIE L*a*b* Color Scale: http://cobra.rdsor.ro/cursuri/cielab.pdf

[2] ICtCp Color Space: http://www.dolby.com/us/en/technologies/dolby-vision/ICtCp-white-paper.pdf

[3] Society of Motion Picture & Television Engineers, "High Dynamic Range Electro-Optical Transfer Function of Mastering Reference Displays", SMPTE ST 2084:2014, August 2014, 1-14.

[4] Pytlarz, J., Pieri, E., Atkins, R. 2017. Objectively Evaluating High-Dynamic-Range and Wide-Color-Gamut Color Differences. SMPTE Motion Imaging Journal. March 2017, pp. 27 to 32.

[5] International Telecommunication Union (ITU), 2017. Image parameter values for high dynamic range television for use in production and international programme exchange, Recommendation ITU-R BT.2100-1.

[6] Zhang, Chen. Efficient Feature Extraction for 2D/3D Objects in Mesh Representation

[7] International Telecommunication Union (ITU), 2015. Parameter values for the HDTV standards for production and international programme exchange, Recommendation ITU-R BT.709-6, 2015.

[8] Society of Motion Picture & Television Engineers, "SMPTE Standard for D-Cinema Quality – Screen Luminance Level, Chromaticity and Uniformity", SMPTE 431-2-2006.

[9] MacAdam, D. L.,1942. Visual Sensitivities to Color Difference in Daylight, Journal of the Optical Society of America, 32, May 1942, pp. 247-274.

CHANGES TO NOTE:

Two changes have been made for this version:

1) The scalars have been changed from [2048,1024,2048] to [720,360,720]. This was to better reflect subjectively determined JND's.

2) The MDC values and plots have been changed to reflect the new scalars given in (1). Compared to Version 5, each MDC value is now scaled by the ratio of $(720/2048)^3$.

APPENDIX A

IC_TC_P Overview:

The IC_TC_P color representation was designed to describe perceptually-uniform just-noticeable differences (JNDs) over a wide range of luminance levels and colors. Then, for efficient encoding, it was stretched to fill the [-0.5, 0.5] and [0, 1] range. Unlike CIE L*a*b*, CIECAM02, or many other color appearance models, it is not normalized by the peak luminance or an estimate of viewer adaptation. Instead, it is an absolute color representation used to describe the number of visually-distinguishable colors that can be presented by a display device.

As shown in the left-hand figure below, the I (or Intensity) channel corresponds closely to luminance encoded with the perceptual quantizer (PQ). Thus, we consider unique steps along the Intensity axis to be a JND. The opponent color axes C_T and C_P are also designed to yield equal JNDs before being stretched to fill the [-0.5, 0.5] range. This is shown in the right-hand figure below using the MacAdam ellipse data [9]. A perfectly uniform color representation yields circular and equal size ellipses and the IC_TC_P color representation exhibits fairly consistent sizes and shapes.
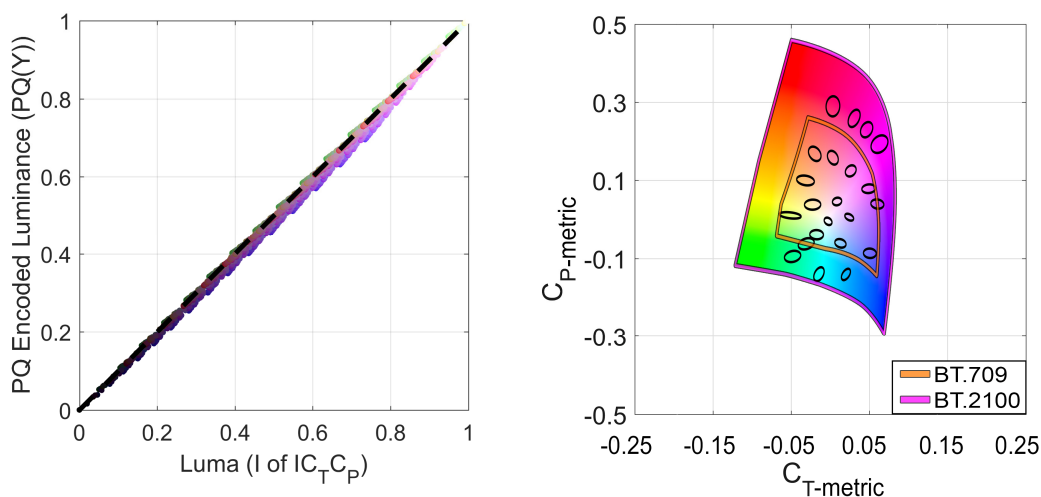

There is no upper limit to the color volume that can be represented by IC_TC_P, because it can describe all the colors within the spectral locus with no limit on the maximum luminance. The MDC scaling is computed such that a value of one in IC_TC_P is roughly a JND. Evaluating the color volume of a display using the IC_TC_P color representation provides a rough count of the number of visually distinguishable colors that the display can reproduce.

Comparing IC_TC_P and CIE L*a*b*:

Below is an example of computing MDC using IC_TC_P and CIE L*a*b*. Using IC_TC_P (an absolute color representation), the display with a 600 cd/m² white level (matched black level and color primaries), as expected, has roughly twice the MDC of the 100 cd/m² display.

IC_TC_P Color Volume (600cd/m²) — 15 MDC

IC_TC_P Color Volume (100cd/m²) — 7 MDC
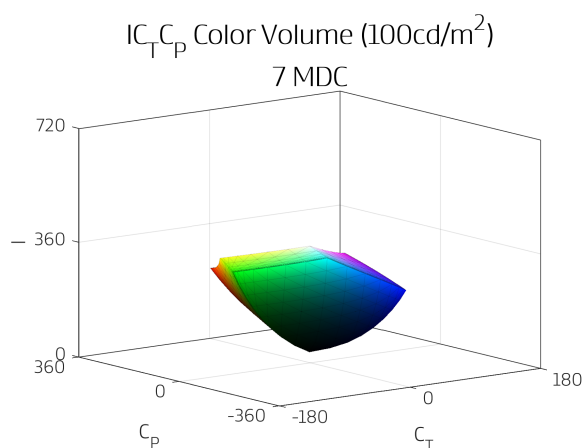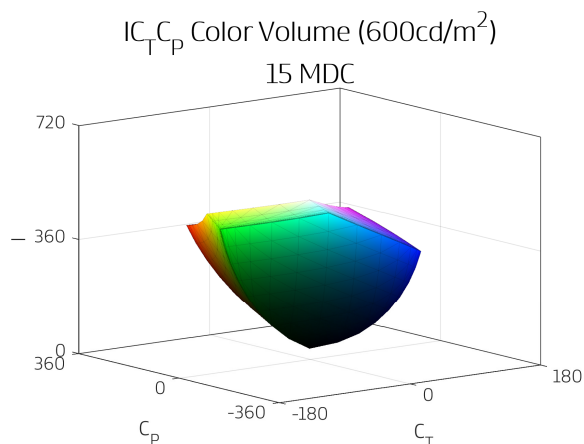
Using CIE L*a*b* (a relative color representation), due to the normalization by peak white, the display with a 600 cd/m² white level has nearly the same MDC as the 100 cd/m² display.

CIE L*a*b* Color Volume (600cd/m²) — 1.2296 MDC

CIE L*a*b* Color Volume (100cd/m²) — 1.2196 MDC
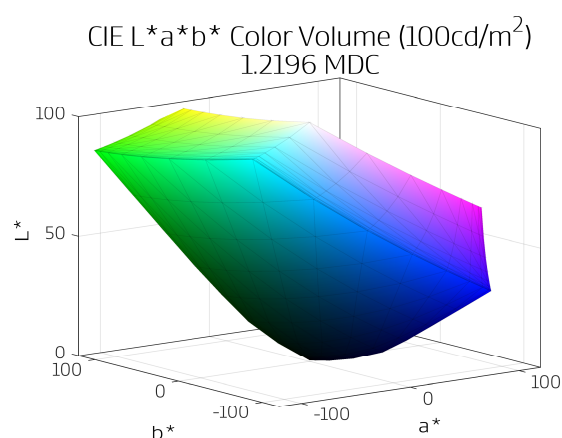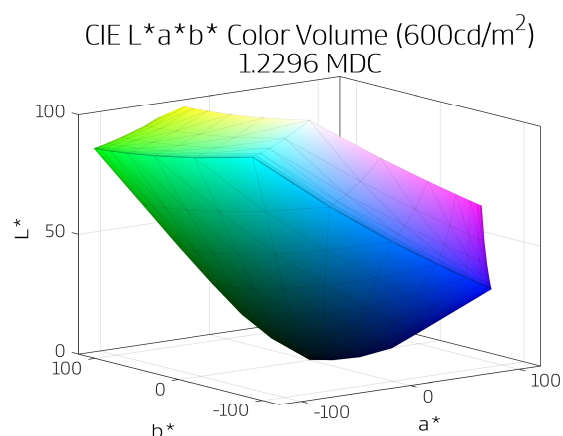
This is the reason why we recommend computing MDC in IC_TC_P. By using an absolute color space, the color reproduction capabilities of different displays can be compared against each other, which is a critical task for marketing, reviewing, and designing display devices.

APPENDIX B

Below is example Matlab code for computing millions of distinguishable colors (MDC).

```matlab
% Define EOTF functions
EOTF = @(PQ) (max(PQ.^(32/2523)-(3424/4096),0) ./ ...
              ((2413/128)-(2392/128)*PQ.^(32/2523))).^(16384/2610)*10000;
invEOTF = @(Lin) (((3424/4096)+(2413/128)*(max(0,Lin)/10000).^(2610/16384)) ./ ...
              (1+(2392/128)*(max(0,Lin)/10000).^(2610/16384))).^(2523/32);
% Define lattice points
N = 9;
Nodes = linspace(0,1,N);
[C1,C2,C3] = meshgrid(Nodes,Nodes,[0 1]);
RGB = [C1(:) C2(:) C3(:); C2(:) C3(:) C1(:); C3(:) C1(:) C2(:)];
% Simulate XYZ measurements
Black = 0.1;
White = 100;
M = [3.2410 -0.9692 0.0556; -1.5374 1.8760 -0.2040; -0.4986 0.0416 1.0570];
XYZ = EOTF(RGB * (invEOTF(White) - invEOTF(Black)) + invEOTF(Black)) / M;

%% Step 2 - CONVERTING TO A PERCEPTUALLY UNIFORM COLOR REPRESENTATION

% Convert to ICtCp
XYZ2LMSmat = [0.3593 -0.1921 0.0071; 0.6976 1.1005 0.0748; -0.0359 0.0754 0.8433];
LMS2ICTCPmat = [2048 2048 0; 6610 -13613 7003; 17933 -17390 -543]'/4096;
ICTCP = bsxfun(@times, invEOTF(XYZ * XYZ2LMSmat) * LMS2ICTCPmat, [720, 360, 720]);

%% Step 3 - COMPUTING THE 3D VOLUME – Millions of Distinguishable Colors (MDC)

% Create tessellation of the surface
[i,j,f] = meshgrid(0:N-2, 0:N-2, 0:5);
startidx = f*N*N + i*N + j + 1;
tri = [[startidx(:), startidx(:)+1, startidx(:) + N+1]; ...
       [startidx(:), startidx(:)+N, startidx(:) + N+1]];
% Calculate the volume of the tessellation
v321 = ICTCP (tri(:,3),1) .* ICTCP (tri(:,2),2) .* ICTCP (tri(:,1),3);
v231 = ICTCP (tri(:,2),1) .* ICTCP (tri(:,3),2) .* ICTCP (tri(:,1),3);
v312 = ICTCP (tri(:,3),1) .* ICTCP (tri(:,1),2) .* ICTCP (tri(:,2),3);
v132 = ICTCP (tri(:,1),1) .* ICTCP (tri(:,3),2) .* ICTCP (tri(:,2),3);
v213 = ICTCP (tri(:,2),1) .* ICTCP (tri(:,1),2) .* ICTCP (tri(:,3),3);
v123 = ICTCP (tri(:,1),1) .* ICTCP (tri(:,2),2) .* ICTCP (tri(:,3),3);
v = abs((1/6)*(-v321 + v231 + v312 - v132 - v213 + v123));
MDC = round(sum(v)/1000000,1);
percHDR = round(MDC/43*100);
percSDR = round(MDC/5.1*100);

%% Optional PLOT THE VOLUME

figure(1), clf; view([-37.5 15]);
patch('faces',tri,'vertices', ICTCP(:,[2 3 1]),...
    'facevertexcdata',(EOTF(RGB)/10000).^(1/3),...
    'facecolor','interp','edgealpha',0.1);
xlabel('C_T'),ylabel('C_P'),zlabel('I'); title(sprintf('Color Volume: %.2g MDC',MDC));
xlim([-720/4 720/4]), ylim([-720/2 720/2]), zlim([0 720]), grid on; box on;
```